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SIMILARITY AND THE ENERGY DISTRIBUTION IN AN EXPLOSION IN AN ELASTIC–PLASTIC MEDIUM

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UDC 539.3

An exact solution of the problem of an explosion in a solid medium where large strains occur is possible by using numerical methods [1, 2]. Results of computations of separate versions of strong explosions are presented in [3-9]. The spherically symmetric explosion is investigated in a medium which differs minimally in complexity of the description from an elastic medium but an important property of a medium subjected to large strains, the capacity to plastic flow, is taken into account for a detailed analysis and to obtain general regularities in this paper. Such an ideal elastic–plastic medium differs from the elastic by one excess parameter, the yield point. The problem of an explosion in such a medium was approximately solved earlier for simplifying assumptions, and a detailed survey is found in [10-14].

The equations of motion continuity and energy in Lagrange variables for the nonstationary motion of a continuous medium with spherical symmetry have the form

$$\frac{\rho_0}{V}\frac{\partial v}{\partial t}=\frac{\partial \sigma_r}{\partial r}+2\frac{\sigma_r-\sigma_\varphi}{r},$$

$$\frac{1}{V}\frac{\partial V}{\partial t}=\frac{1}{r^2}\frac{\partial}{\partial r}(r^2 v),\quad V=\frac{\rho_0}{\rho},$$

$$\rho_0\frac{\partial E}{\partial t}=-p\frac{\partial V}{\partial t}+V\left(S_r\frac{\partial e_r}{\partial t}+2S_\varphi\frac{\partial e_\varphi}{\partial t}\right),$$

$$\frac{\partial e_r}{\partial t}=\frac{\partial v}{\partial r},\ \frac{\partial e_\varphi}{\partial t}=\frac{v}{r},\quad \sigma_r=-p+S_r,\ \sigma_\varphi=-p+S_\varphi,$$

where v is the velocity, ρ is the density of the medium, ρ_0 is the initial density, p is the pressure, σ_r and σ_φ are the radial and tangential stresses, S_r and S_φ are stress deviator components, E is the internal energy of the medium per unit mass, and e_r and e_φ are strain tensor components.

The relationships between the stresses and strains for an elastic material are used in the form

Moscow. Translated from Zhurnal Prikladnoi Mekhaniki i Tekhnicheskoi Fiziki, No. 2, pp. 143–152, March-April, 1980. Original article submitted April 9, 1979.

 0021-8944/80/2102-0276$07.50

$$\frac{\partial S_r}{\partial t} = 2\mu \frac{\partial e'_r}{\partial t}, \quad \frac{\partial S_\varphi}{\partial t} = 2\mu \frac{\partial e'_\varphi}{\partial t},$$
$$\frac{\partial e'_r}{\partial t} = \frac{\partial e_r}{\partial t} - \frac{1}{3V}\frac{\partial V}{\partial t}, \quad \frac{\partial e'_\varphi}{\partial t} = \frac{\partial e_\varphi}{\partial t} - \frac{1}{3V}\frac{\partial V}{\partial t}, \tag{1}$$

where μ is the shear modulus, and $\acute{e}_r$, $\acute{e}_\varphi$ are the strain deviator components. These equations are obtained by differentiating Hooke's law and using the continuity equation.

It is assumed that plastic flow sets in upon compliance with the Mises condition which has the following form for spherical symmetry

$$|\sigma_r - \sigma_\varphi| = \sqrt{3}\cdot kc,$$

where kc is a constant (the yield point under pure shear). Upon compliance with this condition, the Prandtl–Reiss equation

$$2\mu \frac{\partial e'_r}{\partial t} = \frac{\partial S_r}{\partial t} + \lambda S_r, \quad 2\mu \frac{\partial e'_\varphi}{\partial t} = \frac{\partial S_\varphi}{\partial t} + \lambda S_\varphi.$$

should be used in place of (1). The quantity λ is determined from the fluidity condition by the formula

$$\lambda = 0\,(w < 0),\ \lambda = \frac{\mu w}{kc^2}(w \geqslant 0),\ w = S_r \frac{\partial e'_r}{\partial t} + 2S_\varphi \frac{\partial e'_\varphi}{\partial t}.$$

For the constant yield point used in the paper, the quantity w equals the rate of energy dissipation per unit volume in plastic flow.

The case $w < 0$ corresponds to the unloading mode, which occurs elastically. Let us note that an important assumption of the Prandtl–Reiss model about the principal axes of the plastic strain rate tensor being coincident with the principal axes of the stress deviator is satisfied automatically in the case under consideration because of the spherical symmetry of the problem.

The equation of state is taken in the Mie–Grüneisen form with the shear strain energy taken into account [9] since the computations were performed for a medium for which the yield point is of the same order as the elastic moduli

$$p = p_0 + k\eta + k_2\eta^2 + k_3\eta^3 + \rho\Gamma E - I'_2(1+\Gamma)/(2\mu),$$
$$\eta = \rho/\rho_0 - 1,$$

where k is the compression modulus, I'_2 is the second invariant of the stress deviator, Γ is the Grüneisen constant, k_2 and k_3 are constants characterizing the medium, and p_0 is the initial pressure in the medium. In the computations of all the versions presented in this paper, the values of the coefficients in this equation did not vary, quantities with the dimensionality of a pressure were calculated in the units $\rho_0 c_l^2$, where c_l is the longitudinal sound wave velocity,

$$k/(\rho_0 c_l^2) = 0.66,\ \mu/(\rho_0 c_l^2) = 0.25,\ k_2/(\rho_0 c_l^2) = 10^{-3},$$
$$k_3/(\rho_0 c_l^2) = 0.31, \qquad \Gamma = 1.$$

It was assumed that the pressure is constant along the radius in the cavity occupied by the explosion products, and diminishes with increasing radius according to a law represented by two power functions [15]:

$$p = p_p (r_0/r)^{3\gamma_1} (r < r_*),\ \ p = p_* (r_0/r)^{3\gamma_2} \qquad (r \geqslant r_*).$$

The computations were performed for the following values of the constants: $\gamma_1 = 2.81$, $\gamma_2 = 1.26$, $r_*/r_0 = 1.6278$.

The partial differential equations presented were approximated by a difference scheme basically similar to that used in [2] and in [15-17]. This scheme is supplemented in order to make possible the execution of computations for large cavity expansions. An average of such a form was introduced as would operate only on the sawtooth velocity profiles, where the maximum value on the shock front would not be averaged. The scheme being used was not divergent, hence the energy balance of the explosion and the medium were calculated during the computation, on which basis the accuracy of the computations could be estimated. The computation was considered satisfactory if the unbalance did not exceed one percent. To confirm operation of the scheme in the elastic domain, a computation was performed of the problem for which there is an exact analytic solution:

A constant pressure occurs and is sustained in the cavity in the initial instant. Satisfactory agreement is obtained between the numerical and exact solutions in the continuous flow mode (the shock is diffused by artificial viscosity). Let us note that a simple method for reducing the shock amplitude relative to the site on the profile at which the Hugoniot condition is satisfied, is mentioned in [18]. A check on the correctness of scheme operation in the plastic domain was accomplished by a computation of the plastic shock, and a check computation of strong shocks was also performed. The results obtained corresponded to the theoretical relations.

A typical arrangement of the plastic zones and the formation of an elastic wave is shown in [16] for an explosion in an elastic–plastic medium. In the domain of relatively small yield points when the initial cavity is broadened more than twice and in the rated band of the ratio $p_p/(\rho_0 c_l^2)$

$$10^{-4} < kc/p_p < 10^{-2},\ 0.09 < p_p/(\rho_0 c_l^2) < 9. \tag{2}$$

The finite cavity radius r_p and the length of the domain of plastic shock existence l_B can be represented by power-law relationships obtained by processing the results of a numerical computation by least squares:

$$r_p = 0.453 r_0 p_p^{0.284\pm0.002} kc^{-0.301\pm0.002} (\rho_0 c_l^2)^{0.017\pm0.003}; \tag{3}$$

$$l_B = 0.444 r_0 p_p^{0.288\pm0.004} kc^{-0.593\pm0.004} (\rho_0 c_l^2)^{0.305\pm0.005}. \tag{4}$$

The formulas are obtained on the basis of computations in which a compressible medium with Poisson ratio $\nu = 0.33$ was considered. Under these conditions the radius of the cavity being formed depends weakly on the quantity $\rho_0 c_l^2$, i.e., on the compressibility of the medium. In connection with this result, let us note [19, 20] in which an assumption about incompressibility of the medium surrounding the expanding explosion cavity was used as a hypothesis to facilitate the solution. The dependence on the initial pressure was obtained identically in these formulas, within the limits of the errors mentioned. Therefore, similarity of the finite cavity radius and the dimension of the plastic domain exists in explosions in media with the same yield point. This similarity exists not only for finite dimensions but is observed also during a significant time of explosion development, excepting the initial instants. Let us note that the similarity of explosive waves is established for an explosion of charges with different calorific values in air after the shock has passed several radii of the charge [21].

The time development of the cavity radius and the plastic zones is shown in Fig. 1 for explosions in media with four values of the yield point. Each such curve turns out to be identical for explosions with initial pressures differing by hundreds of times and satisfying the inequalities (2). Since the energy of the explosion is proportional to the pressure, then the time of development of such explosions differs greatly. But if the length of the plastic domain l_B is taken in conformity with (4) as the linear scale of the explosion, and the same quantity divided by the elastic wave velocity c_l as the time scale, then explosions with different values p_p are developed identically in such coordinates.

Elastic waves emitted by explosions with different initial pressure also turn out to be similar if they are executed in media with identical yield points. Pressure profiles in emitted elastic waves, computed for four different kc are shown in Fig. 2. The wavelength is referred to the quantity l_B, and the amplitude of the parameter $p_m = \sqrt{3}(1+\nu)kc/(3(1-2\nu))$ which equals the amplitude of the elastic precursor during propagation of a plane plastic shock. This parameter is characteristic even for a spherical wave in which the length of the precursor is not large compared to the distance. In the case under consideration, this length is determined by the difference in the rates of the longitudinal and the so-called volume sound waves c_0. The relative length of the precursor equals the qunatity $(c_l - c_0)/c_0 = \sqrt{3(1-\nu)/(1+\nu)} - 1$. For the Poisson ratio $\nu = 0.33$ used in the computations, it equals 0.23 which is a noticeable magnitude, hence for $r = l_B$ the rated amplitude of the precursor can differ from p_m by approximately the same magnitude. Moreover, the amplitude turns out to be somewhat diminished because of the action of the scheme viscosity. Let us note that asymptotic profiles in the units p/p_m and the medium velocity in the units v/v_m, where $v_m = \sqrt{3}(1-\nu)kc/((1-2\nu)\rho_0 c_l)$ are identical. But the asymptotic profile of the velocity is formed later than the pressure profile, at a distance considerably exceeding l_B where the amplitude of the velocity will be diminished in inverse proportion to the distance.

Let us examine explosions in media with different kc in more detail. According to (3) and (4), the ratio between the length of the plastic domain and the finite cavity size varies in this case; consequently, there is no similarity in explosions in media with different yield points, and explosions develop differently. As is seen from Fig. 1, as the yield point diminishes, the length of the plastic domain around the cavity and its time duration grow. The plastic shock which has a finite width because of the application of artificial viscosity in the computations, is propagated at a constant velocity with the exception of a small section at the cavity, where its velocity will be the higher, the greater the initial pressure in the cavity.

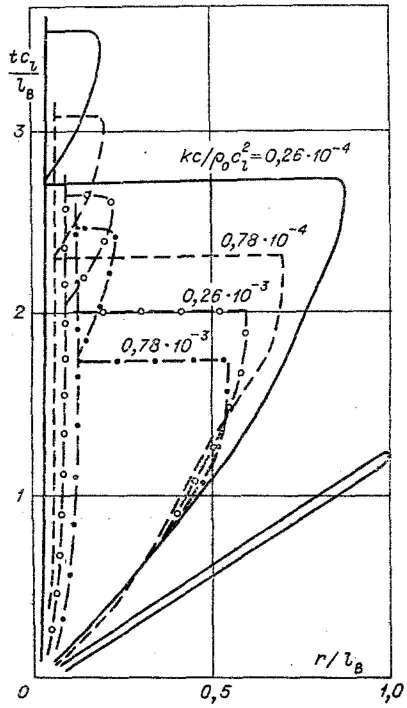

Fig. 1

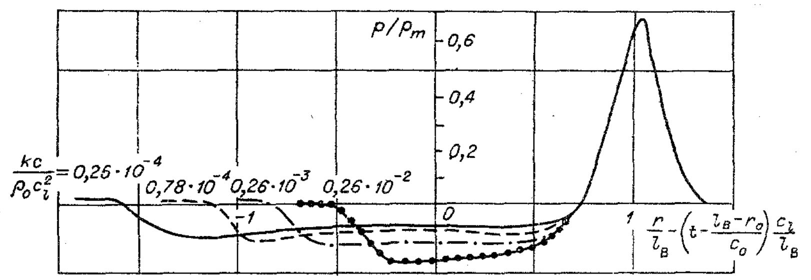

Fig. 2

The positive wave phase turns out to be identical for explosions in media with different yield points in the asymptotic pressure profiles shown in Fig. 2 (in relative coordinates) in the emitted elastic waves. This is explained by the fact that the amplitude is referred to the yield point while the duration of the growing part of the positive phase is not directly dependent on kc but is determined by the magnitude of the difference between c_l and c_0. The amplitude of the negative wave phase diminishes with the diminution in the yield point of the medium, but its duration increases. The plastic domain being formed at the cavity exerts influence on the formation of a negative wave phase. It is seen from a comparison of Figs. 1 and 2 that the increase in the plastic domain around the cavity results in a diminution in the amplitude of the negative wave phase and an increase in its duration. Vibrations of low amplitude coupled to the cavity vibrations still follow the positive and negative wave phases. The first reciprocal motion of the cavity is most intense, and evokes the appearance of still another plastic domain located within the one which had been there earlier for computations with initial data satisfying conditions (2). All the subsequent vibrations are elastic in nature and damp out as in the elastic problem [22]. However, the steady state of the stress near the cavity differs from the stress profile in the purely elastic problem. In an explosion it would occur after the plastic motion and is characterized by the fact that the values of $|\sigma_r|$ and $|\sigma_\varphi|$ grow with distance from the cavity [15], and only start to drop after a certain distance.

The computations executed permitted evaluation of the finite value of the pressure in the cavity p_c. The formula

$$p_c = 1.99 p_p^{-0.063\pm0.005} kc^{1.127\pm0.006} \left(\rho_0 c_l^2\right)^{-0.064\pm0.008} \tag{5}$$

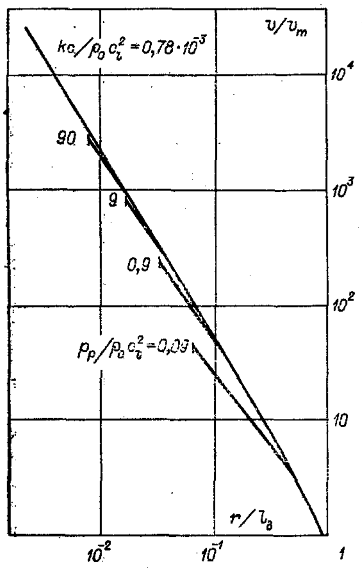

Fig. 3

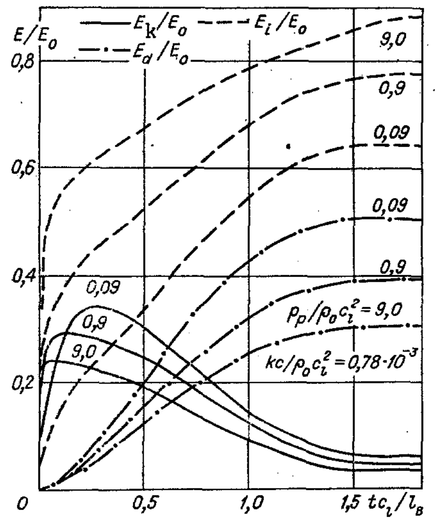

Fig. 4

is obtained in the similarity domain (2). Finally, the steady-state pressure in the cavity is determined mainly by the yield point of the medium. Its dependence on the initial pressure and compressibility of the medium is weak.

The change in amplitude, with distance, of the velocity of the medium in a shock is shown in logarithmic coordinates in Fig. 3 for several versions which differ in the initial pressure in the cavity. The yield point was identical $0.78 \cdot 10^{-3} \rho_0 c_l^2$ in all the versions. The left end of the curves corresponds to the coordinate of the initial change radius. Curves for the lower initial pressure run into the curves for high pressure. This passage is explained by the influence of the initial charge radius. The velocity attenuation lines form a single curve after the shock has traversed approximately three charge radii. At the midsection, the slope of the attenuation lines is 1.6. Computations for other values of kc showed that the slope of the common middle part of the attenuation curve is 1.6. Hence only the position of the initial section changes. As kc changes, l_B increases, hence, the ratio r_0/l_B diminishes, and the beginning of the curve is consequently shifted leftwards.

The velocity amplitude attenuates rather more strongly near the place where the plastic shock vanishes, and the attenuation occurs according to elastic laws at distances exceeding l_B where there is already no shock, although the slope of the curve is initially somewhat greater than one since the wavelength here is not small in comparison to the distance.

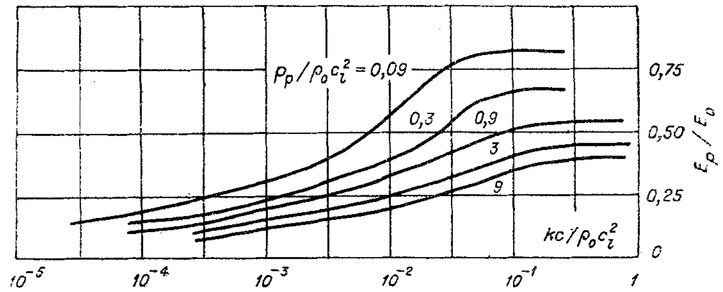

Fig. 5

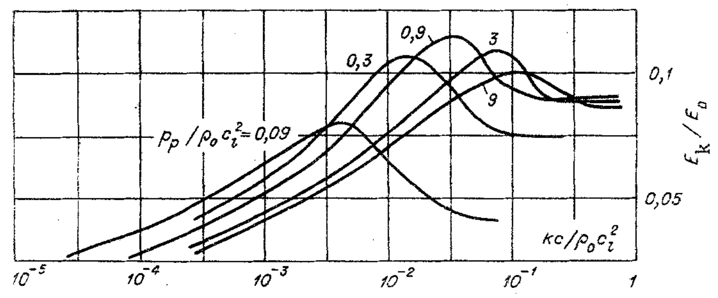

Fig. 6

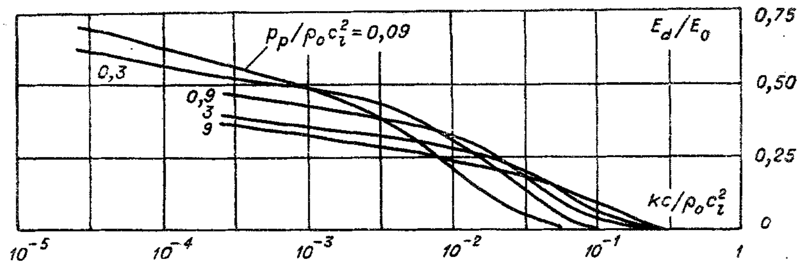

Fig. 7

The time change in the magnitude of the ratio between the kinetic E_k and the internal energies E_i of the medium, as well as of the ratio between the energy dissipated during plastic motion E_d and the explosion energy E_0 is shown in Fig. 4 for three versions in which the initial gas pressure in the cavity differed by hundreds of times. The numbers around the curves show the magnitude of $p_p/(\rho_0 c_l^2)$. The yield point of the medium equals $0.78 \cdot 10^{-3}$ $(\rho_0 c_l^2)$ in all cases. As in Fig. 1, the relative time is plotted along the abscissa axis.

It is seen in Fig. 4 that the higher the initial pressure in the cavity, the more rapidly do the kinetic and internal energies of the medium increase. Energy transfer occurs during expansion of the cavity, whose magnitude at the initial instants is determined by the mass flow rate of the medium in the shock wave that occurs. This velocity is proportional to the pressure in the weak waves considered and to the square root of the pressure in strong waves. Both degrees are greater than the exponent for the pressure in the formula (4) for l_B, hence the explosion energy in the initial instants is transmitted more rapidly to the medium after the same relative time for a large initial gas pressure in the cavity.

The initial internal energy of the medium and the pressure therein were zero in the versions considered. Under such conditions the shock communicates an equal quantity of kinetic and internal energies to the medium; consequently, these kinds of energy agree at the first instants in Fig. 4. Later they develop differently. The internal energy grows monotonically, while the kinetic passes through a maximum. The rather different nature of the relationships between them for a high and low initial pressure in the cavity can also be noted. At the high pressure the internal energy of the medium becomes greater than the kinetic energy. For a low initial pressure, the shock which occurs is weak, and even when it departs sufficiently far, a high pressure still remains in the cavity, hence the cavity continues to expand, thereby increasing the velocity of the medium. In these versions, the kinetic energy exceeds the internal energy up to a certain time. Later the cavity still continues to be broadened and to transmit energy to the medium but more and more slowly because of the adiabatic pressure drop therein. But in this same time the plastic motion already encloses a significant volume, and the dissipation grows so much that it exceeds the additional energy transmitted by the expanding cavity, hence, the kinetic energy of the medium starts to drop. After cessation of the plastic motion, only elastic strain occurs, for which there is no dissipation. The elastic vibrations of the cavity are small and yield no noticeable energy redistribution. The elastic wave is already close to asymptotic form at this time and also does not result in substantial redistribution. Hence, soon after cessation of the plastic motion, the kinds of energy considered retain constant values. It is seen in Fig. 4 that the time of emergence at the constant value is approximately identical for all kinds of energies.

Let us note the singularity of reaching the final distribution of explosion energy in a solid medium. The shock front is either conserved to infinity in an explosion in air or in water if an ideal medium is considered, or is spread by viscous and heat-conductive effects in the weak stage. In both cases there exists a dissipation and continuous diminution of the explosive wave energy which does not cease to infinite time. The energy remaining in the concluding stage of explosion development can be judged only approximately in these media since it is not clear at what time the calculation is performed. Dissipation ceases after the cessation of the plastic flow in an explosion in an elastic–plastic material. Hence, the ratio between the explosion energy and those kinds of energies between which it is distributed can be evaluated perfectly definitely in such a medium.

The final value of the gas energy in an expanding cavity is shown in Fig. 5. This energy grows as the ratio $kc/(\rho_0 c_l^2)$ increases since the greater the yield point, the less does the cavity expand, and the greater is the energy remaining therein. The horizontal sections on the right sides of the curves correspond to elastic motion of the medium. Each curve corresponds to a definite value of the initial gas pressure in the cavity, which is indicated near the line. The numbers denote the ratio $p_p/(\rho_0 c_l^2)$. The diminution in the energy E_p remaining in the cavity with the rise in the initial pressure is explained as follows. The ratio E_p/E_0 is proportional to the quantity $p_c V_f / p_p V_0$, where V_0 and V_f are the initial and volumes of the cavity. Hence, by using (3), (5), we obtain $E_p/E_0 \sim p_p^{-0.21}$. The main reason for the existence of the considered reciprocal regularity is that the degree of the dependence of the final cavity radius on the initial pressure in (3) is less than 1/3.

The final value of the kinetic energy of the medium is shown in Fig. 6. The notation is the same as in Fig. 5. A characteristic singularity is the presence of maximum energy for a change in the ratio $kc/(\rho_0 c_l^2)$. As the initial pressure changes in the cavity but the yield point is unchanged, a maximum is also obtained. In the similarity domain (2) the kinetic energy diminishes monotonically as the initial pressure in the cavity increases. This pressure dependence can be obtained as follows. The relative pressure profiles which agree with the relative asymptotic profiles of the velocity of the medium are shown in Fig. 2. Integrating the square of the velocity over the profiles mentioned, each of which is the same for constant kc but different p_p because of the similarity noted, and taking into account that the distance on the graph is divided by l_B, we obtain $E_k \sim l_B^3$, i.e., the final value of the kinetic energy is proportional to the volume of the plastic domain. Using (4), we obtain $E_k/E_0 \sim p_p^{-0.14}$.

The dependence on another parameter, on the yield point of the medium, can be estimated only approximately by this method since the velocity profiles in the negative phase are different for different quantities kc. Nevertheless, such an estimate also results in the fact that the kinetic energy grows in the similarity domain as the yield point increases. Let us note that after the plastic motion ceases, almost all the kinetic energy is concentrated in the elastic zone, its fraction due to motion around the cavity is not large. The kinetic energy equals the elastic energy in the explosion wave studied; therefore, twice the value of the kinetic energy yields the magnitude of the total energy of such a wave.

The final value of the energy dissipated in the plastic motion of the medium is shown in Fig. 7. It drops monotonically as the yield point increases. The magnitude of this energy is approximately proportional to the product kc and the volume of the plastic domain l_B^3 so that by using (4) we obtain the approximate quantity $E_d/E_0 \sim kc^{-0.78}$ in the similarity domain (2). Outside the domain (2), dissipation also diminishes as kc increases and vanishes in the absence of plastic motion.

The change in p_p also exerts influence on the magnitude of the energy under consideration. For large values of kc the energy dissipation drops as p_p diminishes, and vanishes for a definite initial pressure when it is inadequate for the occurrence of plastic motion. For relatively small values of kc in the similarity domain (2), there is obtained that the greater the initial pressure in the cavity, the smaller the fraction of explosion energy dissipated in plastic motion. However, the shock intensity and the associated irreversible energy losses hence grow because of the growth of entropy in the shock. As computations show, the total losses grow with the increase in initial pressure in the cavity.

In conclusion, we examine the question of the effectiveness of an explosion. An estimate of this property depends on what is taken as useful work. If the explosion is considered as a source of elastic waves, then the energy of the emitted wave should be taken as the useful energy. For typical values of $kc/(\rho_0 c_l^2) = 10^{-3}$ and $p_p/(\rho_0 c_l^2) = 1$, about 10% of the explosion energy goes over into the elastic wave. If the explosion is used to produce a cavity, then its formation is certainly accompanied by large strains, which are possible only during plastic motion. Hence, the energy being dissipated in plastic motion around the cavity should be referred to useful work. For typical parameters this is about 40% of the explosion energy.

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